Spatial prisoner's dilemma games with dynamic payoff matrices

Masaki Tomochi[1] and Mitsuo Kono[2]
[1]Institute for Mathematical Behavioral Sciences, University of California, Irvine, 3151 Social Sciences Plaza, Irvine, California 92697
[2]Faculty of Policy Studies, Chuo University, Hachioji, Tokyo 192-0393, Japan

(Received 26 March 2001; revised manuscript received 4 September 2001; published 16 January 2002)

The effects of dynamic payoff matrices on evolution of cooperation are studied based on the prisoner's dilemma game on a two-dimensional square lattice. The study is conducted by simulation and an analytical theory based on mean-field approximation. Payoff matrices are designed to evolve depending on a ratio of defectors (or cooperators) to the whole population. Dynamic payoff matrices are necessary to describe evolution of a society whose payoff may be affected by the results of actions of the members in the society. Introducing such payoff matrices helps to model dynamic aspects of societies.

DOI: 10.1103/PhysRevE.65.026112 PACS number(s): 02.50.Le

I. INTRODUCTION

Human beings may be regarded as players who live in either physically or abstractly limited territory where they interact more frequently with their neighbors than with those who are far away. Their success generally depends on whether they can get along with their neighbors, who often play special roles by showing how to succeed in their lives. It is commonly observed that people try to imitate a strategy of their most successful neighbor. As a result, successful strategies spread from neighbor to neighbor throughout a territory. This is one of the basic ideas behind the evolutionary prisoner's dilemma games that have been studied to analyze phenomena of evolution of cooperation [1–3]. Since Axelrod [2] first suggested and Nowak, May, Bohoeffer, and others [4–15] extended the ideas of the prisoner's dilemma games on a lattice, spatial prisoner's dilemma games have received much attention. In spatial prisoner's dilemma games, it is possible to observe coexistence and coevolution between cooperation and defection mainly depending on elements of payoff matrices of the prisoner's dilemma [4–9,11,12]. It has been shown that, in the spatial prisoner's dilemma game, clusters of cooperators play an important role in the evolution of cooperation [1,2,4–9,11–13].

In this paper, dynamic payoff matrices are incorporated into the ideas of spatial prisoner's dilemma games to model dynamic aspects of societies. It is natural to consider that payoff in a society may be affected by the results of actions of the members in the society and could be described by a ratio of defectors (or cooperators) to the whole population. Therefore, payoff matrices are designed to evolve depending on a ratio of defectors (or cooperators) to the whole population.

In the next section, rules of the game are explained. In Sec. III, evolution of the society is shown to depend on magnitudes of payoff-matrix elements under the condition that the prisoner's dilemma is defined. These results lead to an introduction of dynamic payoff matrices and, in Sec. IV, we study how the dynamic payoff matrices affect evolution of cooperation. In Sec. V, an analytical theory based on mean-field approximation is formulated to describe a feature of the results of the simulation in Sec. IV. Discussions are given in the last section.

II. RULES OF THE GAME

Following the way of formalization utilized by Herz [11], rules of the game in this paper are defined as follows. Players are placed on a two-dimensional square lattice. The size of the population is given by N, and each player is labeled by i where $1 \leqslant i \leqslant N$. Each player i plays a one-shot prisoner's dilemma game with its local neighbors, excluding the player i itself, denoted by $\tilde{n}(i)$.

The payoff matrix of the prisoner's dilemma game between players i and j is given in Table I where the following inequalities should hold due to the definition of the prisoner's dilemma:

$$S < P < R < T. \tag{1}$$

Because each player plays the one-shot prisoner's dilemma game, the strategy of player i at a unit time t denoted by $\sigma_i(t)$ is either to cooperate or defect,

$$\sigma_i(t) = \begin{cases} +1 & \text{if player } i \text{ cooperates} \\ -1 & \text{if player } i \text{ defects.} \end{cases} \tag{2}$$

The payoff function for player i in a game with player j can be denoted as $f(\sigma_i(t), \sigma_j(t))$, that is, $f(+1,+1) = R$, $f(+1,-1) = S$, $f(-1,+1) = T$, and $f(-1,-1) = P$. The score of player i at time t, $u_i(t)$, is defined as the sum of the outcomes of the games with its local neighbors,

$$u_i(t) = \sum_{j \in \tilde{n}(i)} f(\sigma_i(t), \sigma_j(t)). \tag{3}$$

The updating rule adopted in this paper is so called "copy cat," therefore, player i's strategy at time $t+1$ is defined as follows:

TABLE I. Payoff matrix of the prisoner's dilemma game [see inequalities (1)].

	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

$$\sigma_i(t+1) = \mathrm{sgn}[\max_{j\in c(i)}\{u_j(t)\} - \max_{j\in d(i)}\{u_j(t)\}], \tag{4}$$

where $\mathrm{sgn}[0]=\sigma_i(t)$ is assumed, and $c(i)$ and $d(i)$ stand for cooperators and defectors, respectively, in $n(i)$ that includes both $\tilde{n}(i)$ and i itself. In copy cat, each player imitates a strategy of its most successful neighbor in terms of players' scores. Copy cat is adopted in this paper because, as mentioned in the preceding section, it is commonly observed that people try to imitate a strategy of their most successful neighbor.

The updating described above occurs with a probability $\mu(1/N \leqslant \mu \leqslant 1)$, which denotes a degree of synchronous updating [9,14]. During each period, players update their previous strategy with a probability μ and skip to update with a probability $1-\mu$. As μ goes to 1, the updating becomes perfectly synchronous updating, while to $1/N$, completely asynchronous updating, which was proposed by Huberman and Glance [10]. Introducing the parameter μ allows us to see an effect of stochastic nature in decision making on evolution of cooperation.

III. NUMERICAL RESULTS OF PAYOFF-MATRIX DEPENDENCE

Numerical experiments are performed based on the rules described in the preceding section. We can let $S=0$ and $T=1$ without loss of generality because relative magnitudes of elements of the payoff matrix can be kept. The variables R and P are now treated as parameters. Then the inequalities (1) read as

$$0<P<R<1. \tag{5}$$

We examine payoff-matrix dependence on the evolution of cooperation in our parameter space [4–9]. In the following, the number of neighbors (n_0), which is invariant in i, is set as 8, that is, the Moore neighborhood is used.

In order to microscopically see how defectors spread in a society of cooperators, we put a single defector in the center cell of the game field and cooperators in the remaining cells. For the moment, μ is set as 1. Typical examples of the evolution for the cases of $\{P, R\}$ as (i) {0.2,0.6}, (ii) {0.4,0.6}, (iii) {0.2,0.8}, and (iv) {0.1,0.8} are examined. In the cases of (i) and (ii), defectors are observed to keep spreading in a society of cooperators, which is similar to those found in Axelrod's computer experiments at an initial instance [2]. On the other hand, in the case of (iii), defectors cannot keep spreading out over the society and is confined to a limited area. However, this does not imply that the society is free from being occupied by defectors, since usually there is more than one defector initially. In the case of (iv), interestingly enough, defectors cannot really spread and sometimes even shrink.

The parameter space of $\{P,R\}$ specified for the case of (iv) can be obtained with the aid of Fig. 1 where scores of the players near the center of the game field at the second generation are depicted and cells occupied by defectors are painted gray, otherwise painted white. Four defectors at the second generation in (iv) are converted into cooperators at the third generation, and it continues periodically. The condition for those four defectors to locate at the corner of the cluster consisting of nine defectors is given by the following inequality:

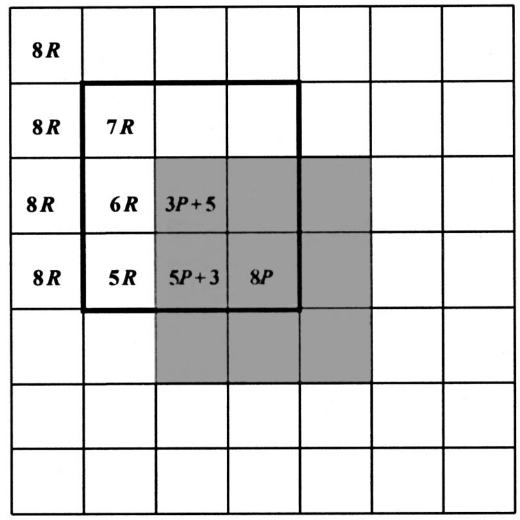

FIG. 1. The parameter space for $\{P, R\}$ for the case of (iv) can be obtained where scores of the players near the center of the game field at the second generation are depicted. Cells occupied by defectors are painted gray, otherwise painted white.

$$\max_{j\in c(i)}\{u_j(t)\} \geqslant \max_{j\in d(i)}\{u_j(t)\} \Rightarrow 7R \geqslant 3P+5. \tag{6}$$

When parameter space of payoff elements $\{P,R\}$ is given by Eqs. (5) and (6), which we refer to as the region I and the remaining region as the region II, cooperators cannot be overwhelmed by defectors because, with these payoff elements, most likely Eq. (4) gives +1 in the whole population; therefore, defectors cannot keep spreading. Figure 2 shows the evolution of cooperation in terms of a ratio of cooperators to the whole population, $p_c(t)$, when μ is set as 1.0, 0.5, and 0.1. Here, the number of the population N is set as 101^2. Payoff elements $\{P,R\}$ are given as (a) {0.1,0.8} and (b) {0.2,0.6}, and the initial ratio between cooperators and defectors is 1:1. In Fig. 2(a), even though it takes longer time for the system to reach its stable state as μ decreases, the majority of the population turns to be cooperators sooner or later because $\{P, R\}=\{0.1,0.8\}$ are in the region I. When μ is set as 1, the stable state is periodic (but not always periodic) because there exist some blinkers in the game field. On the other hand, in Fig. 2(b), the majority of the population eventually becomes defectors when $\{P,R\}$ is {0.2,0.6}, which is in the region II. Additionally, from Fig. 2, one can say that the effect of degrees of synchronous updating on evolution of cooperation (and defection) is not crucial, and this result agrees with the one obtained by Nowak, Bohoeffer, and May [6,7]. In a game field corresponding to Fig. 2(a), it can be observed that clusters of cooperators are expanding gradually among the population (figure not shown) [4–9].

IV. EFFECTS OF DYNAMIC PAYOFF MATRICES ON EVOLUTION OF COOPERATION

As was shown in the preceding section, when $\{P,R\}$ is in region I, the majority of the members in the society be-

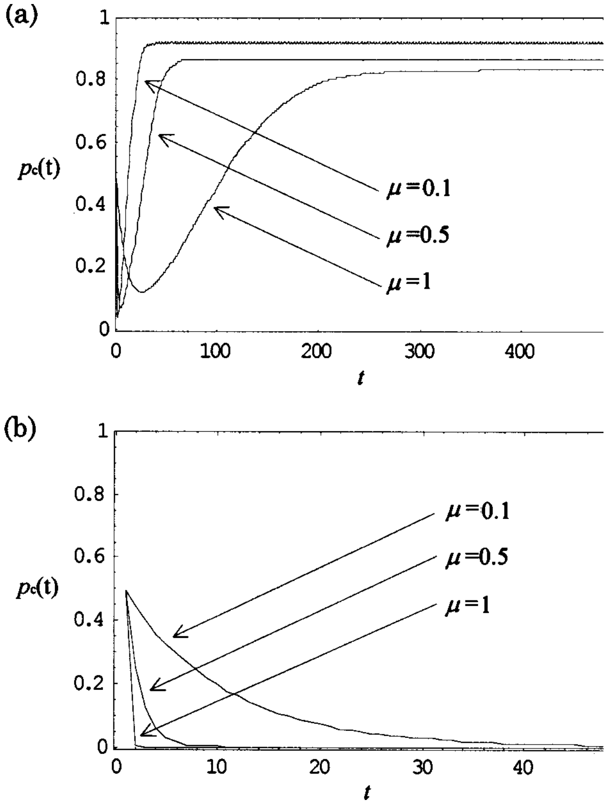

FIG. 2. Evolution of cooperation in terms of a ratio of cooperators, $p_C(t)$, in the whole population N when μ is set as 1.0, 0.5, and 0.1. Payoff-matrix elements $\{P,R\}$ are chosen as (a) $\{0.1,0.8\}$ and (b) $\{0.2,0.6\}$, and the initial ration between cooperators and defectors is 1:1. N is set as 101^2 and t represents a unit time.

come cooperators, but after a certain point at which the system reaches equilibrium, further evolution of cooperation (and defection also) is no longer observed. On the other hand, when the payoff-matrix elements, $\{P,R\}$, are in region II, the majority always defect when fixed payoff matrices are adopted. Then the question arises whether the society acts to adjust its organization before it is filled with defectors. This question is related to the tragedy of the commons, which is certainly observed in real society but may not stay forever [16]. A fair society sooner or later responds to unfavorable changes, introducing some kinds of social reformation that are incorporated into a change of payoff matrices. In any cases with fixed payoffs, after a certain time the system stops evolving.

It is considered that payoffs in a society may be affected by the results of actions of the members in the society and that the situation may be described by a ratio of defectors (or cooperators) to the whole population. Here we describe a model with dynamic payoff elements of the prisoner's dilemma game where S, P, R, and T are set as 0, $P(t)$, $R(t)$, and 1, respectively, and therefore, the following inequality:

$$0<P(t)<R(t)<1. \tag{7}$$

We assume that the payoff-matrix elements $\{P(t),R(t)\}$ change with time t as follows:

$$P(t+1)=P(t)-kg(p_D(t)-p_D^*) \text{ and } \tag{8}$$

$$R(t+1)=R(t)+k'g(p_D(t)-p_D^*), \tag{9}$$

where $g(\rho)$ is an arbitrary function, and $p_D(t)$ is a ratio of the defectors at time t, and therefore, $p_C(t)+p_D(t)=1$ holds. The fixed value, p_D^*, is a critical value of the ratio of the defectors above which reward for cooperators and punishment for defectors are increased and below which opposite effects are taken to the payoff elements. Note that a degree of punishment increases when the value of $P(t)$ decreases. The k and k' control how much feedback of $g(\rho)$ is taken into account on the payoffs and are chosen equal to each other in this paper. For simplicity, we require $P(t)+R(t)$ to be kept constant. In fact, Eqs. (8) and (9) with $k=k'$ give

$$P(t+1)+R(t+1)=P(t)+R(t)=\cdots=P(1)+R(1). \tag{10}$$

In the following, we choose $g(\rho)$ to be a simple linear function

$$g(\rho)=\rho, \tag{11}$$

and set $\{P(1),R(1)\}$, $p_D(1)$, and p_D^* as $\{0.2,0.8\}$, 0.5, and 0.5, respectively. Typical examples of evolution of cooperation in terms of a ratio of cooperators to the whole population, $p_C(t)$, and payoff elements, $P(t)$ and $R(t)$, are shown in Fig. 3 where $k(=k')$ is chosen as 0.01 and μ is fixed as (a) 1.0, (b) 0.5, and (c) 0.1. N is set as 101^2.

The characteristic feature of the evolution of cooperation is intermittent sudden jump ups and downs. Especially, sudden jump ups occur shortly after a set of the payoff-matrix elements $\{P(t),R(t)\}$ crosses the boundary between the regions I and II. Note that $P(t)$ and $R(t)$ always satisfy inequalities (7), that is, the game is always given as the prisoner's dilemma even though payoffs are dynamically changing. The frequencies of jumps depend on the magnitude of $k(=k')$, though the time evolution is not exactly periodic. Sharp transitions between higher and lower values of the ratio of cooperators are a result of accumulation of changes in $P(t)$ and $R(t)$. The p_D^* controls a position of the center where oscillation in evolution of cooperation are taking place. Figure 4 shows snapshots of a typical example of the game field when a sudden jump up is observed in the case of Fig. 3(b) from $t=324$ to 328. It is observed that the edges of clusters of defectors are eroded within a short period. The sudden erosions in the game field in Fig. 4 correspond to the sudden jumps in a value of $p_C(t)$ in Fig. 3.

Thus with dynamic payoff-matrix elements controlled in a self-organized way, society shows a potential ability to return to one dominated by cooperators. However, the society is not able to expel defectors completely, leaving a space for free riders to develop, and repeats a similar cycle, reflecting the fact that history repeats itself.

V. MEAN-FIELD THEORY

In this section, an analytical theory based on mean-field approximation is formulated to recover the results of the

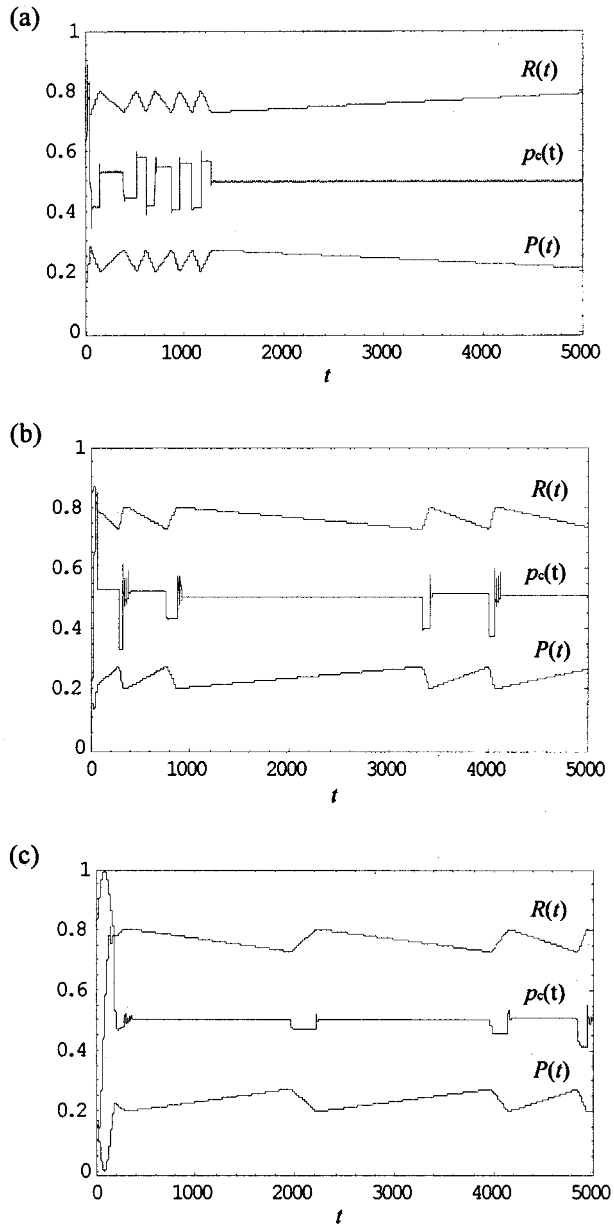

FIG. 3. Typical examples of evolution of cooperation in terms of a ratio of cooperators, $p_C(t)$, and payoff elements, $P(t)$ and $R(t)$, where k $(=k')$ is chosen as 0.01 and μ is fixed as (a) 1.0, (b) 0.5, and (c) 0.1. $\{P(1),R(1)\}$, $p_D(1)$, and p_D^* are given as {0.2,0.8}, 0.5, and 0.5; respectively. N is set as 101^2 and t represents a unit time.

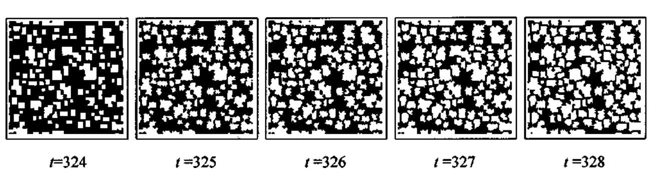

FIG. 4. Snapshots of a typical example of the game field when a sudden jump up is observed in the case of Fig. 3(b) from $t=324$ to 328. Black and white represents defectors and cooperators, respectively. It is observed that the edges of clusters of defectors are eroded within a short period. The sudden erosions in the game field in Fig. 4 correspond to the sudden jumps in a value of $p_C(t)$ in Fig. 3.

simulation in the preceding section. The score of the i player with the strategy $\sigma_i(t)$ at time t, $u_i(\sigma_i(t))$ is given by the sum of the scores obtained by playing games with all the nearest neighbors:

$$\begin{aligned} u_i(\sigma_i(t)) &= \frac{1}{2}\{[R(t)p_i(\sigma_i(t))+Sq_i(\sigma_i(t))][1+\sigma_i(t)] \\ &\quad +[Tq_i(\sigma_i(t))+P(t)p_i(\sigma_i(t))][1-\sigma_i(t)]\} \\ &= \frac{1}{2}\{[R(t)+P(t)]p_i(\sigma_i(t))+(S+T)q_i(\sigma_i(t)) \\ &\quad -[(P(t)-R(t))p_i(\sigma_i(t)) \\ &\quad +(T-S)q_i(\sigma_i(t))]\sigma_i(t)\}, \end{aligned} \tag{12}$$

where $p_i(\sigma_i(t))$ is the portion of the players with the strategy $\sigma_i(t)$ among the nearest neighbors and is given by

$$p_i(\sigma_i(t))=\frac{1}{2n_0}\sum_{j\in\tilde{n}(i)}[1+\sigma_i(t)\sigma_j(t)], \tag{13}$$

and

$$q_i(\sigma_i(t))=1-p_i(\sigma_i(t)). \tag{14}$$

The player i keeps his strategy at time $t+1$ if his score at time t is the largest among those obtained at time t by his nearest neighbors and otherwise adopts the strategy of the player who obtains the maximum score. Thus the strategy at time $t+1$ is given by

$$\sigma_i(t+1)=\sigma^M_{k\in n(i)}(t), \tag{15}$$

where $\sigma^M_{k\in n(i)}(t)$ is the strategy of the player who obtains the maximum score among the i's nearest neighbors, including the i itself, and $u^M_{k\in n(i)}[\sigma^M_{k\in n(i)}(t)]$ is the corresponding score

$$\begin{aligned} \sigma^M_{k\in n(i)}(t) &= \mathrm{sgn}\{R(t)p^M_{m\in n(i)}(+1,t)+Sq^M_{k\in n(i)}(+1,t) \\ &\quad -Tq^M_{k\in n(i)}(-1,t)-P(t)p^M_{k\in n(i)}(-1,t)\}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} & u^M_{k\in n(i)}[\sigma^M_{k\in n(i)}(t)] \\ &= \max\{R(t)p^M_{k\in n(i)}(+1,t)+Sq^M_{k\in n(i)}(+1,t), \\ &\quad Tq^M_{k\in n(i)}(-1,t)+P(t)p^M_{k\in n(i)}(-1,t)\}, \end{aligned} \tag{17}$$

where $p^M_{k\in n(i)}(\pm 1,t)=p^M_{k\in n(i)}[\sigma^M_{k\in n(i)}(t)=\pm 1]$. Here $p^M_{k\in n(i)}[\sigma^M_{k\in n(i)}(t)]$ is the portion of the neighbors having the same strategy as that of the player with the largest score and is given by

$$p^M_{k\in n(i)}[\sigma^M_{k\in n(i)}(t)]=\frac{1}{2n_0}\sum_{j\in\tilde{n}(k)}[1+\sigma^M_{k\in n(i)}(t)\sigma_j(t)]. \tag{18}$$

From Eqs. (13) and (15), time evolution of the portion of the neighboring players with the same strategy as the i player has is given by

$$\begin{aligned}p_i(\pm 1,t+1)-p_i(\pm 1,t)&=\frac{\pm 1}{2n_0}\sum_{j\in\tilde{n}(i)}[\sigma_j(t+1)-\sigma_j(t)]\\&=\frac{\pm 1}{2n_0}\sum_{j\in\tilde{n}(i)}[\sigma^M_{k\in n(j)}(t)-\sigma_j(t)],\end{aligned} \tag{19}$$

where $p_i(\pm 1,t)=p_i(\sigma_i(t)=\pm 1)$.

Then Eq. (19) is rewritten as

$$\begin{aligned}&p_i(\pm 1,t+1)-p_i(\pm 1,t)\\&\quad=\frac{\pm 1}{2n_0}\left\{\sum_{j\in\tilde{d}(i)}[\sigma^M_{k\in n(j)}(t)-\sigma_j(t)]+\sum_{j\in\tilde{c}(i)}[\sigma^M_{k\in n(j)}(t)-\sigma_j(t)]\right\}\\&\quad=\frac{\pm 1}{2n_0}\left\{\sum_{j\in\tilde{n}(i)}[1-\sigma_j(t)]\frac{1+\sigma^M_{k\in n(j)}(t)}{2}-\sum_{j\in\tilde{n}(i)}[1+\sigma_j(t)]\frac{1-\sigma^M_{k\in n(j)}(t)}{2}\right\},\end{aligned} \tag{20}$$

where $\tilde{c}(i)$ and $\tilde{d}(i)$ stand for cooperators and defectors, respectively, in $\tilde{n}(i)$.

It is worthwhile to note that the expressions for $p_i(\sigma_i(t))$ and $q_i(\sigma_i(t))$ are symmetric with each other as a result of the conservation

$$p_i(\sigma_i(t+1))+q_i(\sigma_i(t+1))=p_i(\sigma_i(t))+q_i(\sigma_i(t))=1. \tag{21}$$

Here we introduce the global densities of the cooperators and defectors as

$$p_C(t)=\frac{1}{N}\sum_{i\in N}p_i(+1,t),\quad p_D(t)=\frac{1}{N}\sum_{i\in N}q_i(+1,t). \tag{22}$$

Certainly we have

$$p_C(t)+p_D(t)=1. \tag{23}$$

When the field is large enough to allow the local density $p_i(t)$ and $q_i(t)$ to be replaced by the global density $p_C(t)$ and $p_D(t)$, we have the following equations:

$$\begin{aligned}&\frac{1}{N}\sum_{i\in N}[p_i(\pm 1,t+1)-p_i(\pm,t)]\\&\quad=\frac{1}{N}\sum_{i\in N}\left[\frac{\pm 1}{2n_0}\left\{\sum_{j\in\tilde{n}(i)}[1-\sigma_j(t)]\frac{1+\sigma^M_{k\in n(j)}(t)}{2}-\sum_{j\in\tilde{n}(i)}[1+\sigma_j(t)]\frac{1-\sigma^M_{k\in n(j)}(t)}{2}\right\}\right]\\&\quad=\pm\Pr[\sigma^M_{k\in n[j\in n(i)]}=+1|\forall i\in N]\frac{1}{N}\sum_{i\in N}\frac{1}{2n_0}\times\sum_{j\in\tilde{n}(i)}[1-\sigma_j(t)]\mp\Pr[\sigma^M_{k\in n[j\in n(i)]}(t)\\&\quad=-1|\forall i\in N]\times\frac{1}{N}\sum_{i\in N}\frac{1}{2n_0}\sum_{j\in\tilde{n}(i)}[1+\sigma_j(t)],\end{aligned} \tag{24}$$

which lead to

$$p_C(t+1)-p_C(t)=\alpha(t)p_D(t)-\beta(t)p_C(t) \tag{25}$$

and

$$p_D(t+1)-p_D(t)=-\alpha(t)p_D(t)+\beta(t)p_C(t), \tag{26}$$

where $\alpha(t)$ and $\beta(t)$ are transition probabilities with which defectors change their strategy from defect to cooperate and cooperators do in the reverse way, respectively, and therefore approximated as

$$\begin{aligned}\alpha(t)&=\Pr[\sigma^M_{k\in n[j\in n(i)]}(t)=+1|\forall i\in N]\\&\simeq\frac{a}{2}\{1+\mathrm{sgn}[R(t)-R_{\max}]\},\end{aligned} \tag{27}$$

$$\begin{aligned}\beta(t)&=\Pr[\sigma^M_{k\in n(j\in n(i))}(t)=-1|\forall i\in N]\\&\simeq\frac{a}{2}\{1+\mathrm{sgn}[P(t)-P_{\max}]\},\end{aligned} \tag{28}$$

where a $(0<a<1)$ is a parameter that controls the magnitude of the probability and $P_{\max}$ and $R_{\max}$ are values of $P(t)$ and $R(t)$, respectively, for which $p_C(t)$ jumps either from the region I to the region II or in the reverse way. It should be emphasized that the conserved quantity Eq. (23) derived directly from Eq. (21), or Eqs. (25) and (26) guarantee the stability of the evolution of our dynamical system.

Figure 5 describes evolution of $p_C(t)$, $P(t)$, and $R(t)$, which are obtained by solving Eqs. (8), (9), (25), and (26). The parameters k $(=k')$ and a are chosen as 0.01 and 0.03, respectively, and $p_C(1)$, $P(1)$, $R(1)$, and p^*_D are given as 0.35, 0.2, 0.8, and 0.5, respectively. $P_{\max}$ and $R_{\max}$ are set as $P^*+\delta$ and $R^*+\delta$, respectively, where P^* and R^* are obtained as an intersection of the boundary between the regions I and II and the line in Eq. (10). The δ represents a positive small number introduced for initiating dynamic behavior since P^* and R^* are fixed points of the dynamical system and nothing happens for δ=0. Here δ is fixed as 0.03. Figure 5 shows the characteristic feature of the simulation results in Fig. 3 in the previous section, though not exactly

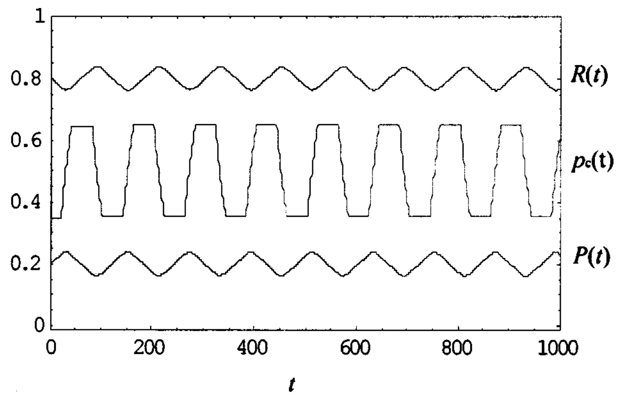

FIG. 5. Evolution of $p_C(t)$, $P(t)$, and $R(t)$, which are obtained by solving Eqs. (8), (9), (25), and (26). The parameters k $(=k')$ and a are chosen as 0.01 and 0.03, respectively, and $p_C(0)$, $P(0)$, $R(0)$, and p_D^* are given as 0.35, 0.2, 0.8, and 0.5, respectively. $P_{\max}$ and $R_{\max}$ are set as $P^* + \delta$ and $R^* + \delta$, respectively, where P^* and R^* are obtained as an intersection of the boundary between the regions I and II and the line in Eq. (10). The δ represents a positive small number introduced for initiating dynamic behavior since P^* and R^* are fixed points of the dynamical system and nothing happens for zero δ. Here δ is fixed as 0.03.

the same due to coarse graining of microscopic individual behavior. Almost same evolution can be obtained if δ is determined by throwing a die.

VI. DISCUSSION

In this paper we have studied effects of dynamic payoff matrices on the evolution of cooperation as well as payoff dependent evolution of cooperation. A modeled society is shown to respond flexibly to its changes if payoff-matrix elements are controlled dynamically. The flexibility of the society is also confirmed by mean-field theory. Often an event that destroys the order of a society subsequently causes a reaction to restore the order. This may be an introduction of some sort of reformation accompanied with a change of value concepts in the society, which reflects a change in the payoff matrix. Thus our model can plausibly explain several important aspects of society.

We assumed that the payoff-matrix elements $\{P(t),R(t)\}$ change with time as Eqs. (8) and (9) when ratio-of-defectors dependent payoff matrices was introduced in Sec. IV. A scenario derived from Eqs. (8) and (9) is considered to correspond to the idea that a society makes an effort to reward cooperators and punish defectors appropriately to solve the dilemma. Dawes [17] argued that it is very costly to change payoffs in the sense that the cost of reward and punishment exceeds the product the society derives from having many people cooperate rather than defect. This second dilemma is partly solved by introducing p_D^*, though p_D^* should be determined self-consistently or empirically, which is our future problem. (And note that the payoff matrices in this paper always satisfy the condition of the prisoner's dilemma even though payoffs are dynamically changing.) A vicious circle of cooperation and defection in the population, which we consider human beings can never avoid, can be observed in our model.

ACKNOWLEDGMENTS

We are obliged to many people for helpful comments and discussion, especially to John Paul Boyd and Hojin Lee.

[1] R. Axelrod and W. D. Hamilton, Science **211**, 1390 (1981).
[2] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).
[3] J. M. Smith, *Evolution and the Theory of Games* (Cambridge University Press, Cambridge, 1982).
[4] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992).
[5] M. A. Nowak and R. M. May, Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 35 (1993).
[6] M. A. Nowak, S. Bohoeffer, and R. M. May, Int. J. Bifurcation Chaos Appl. Sci. Eng. **4**, 33 (1994).
[7] M. A. Nowak, S. Bohoeffer, and R. M. May, Proc. Natl. Acad. Sci. U.S.A. **91**, 4877 (1994).
[8] R. M. May, S. Bohoeffer, and M. A. Nowak, in *Advances in Artificial Life: Third European Conference on Artificial Life*, edited by F. Morán, A. Moreno, J. J. Merelo, and P. Chacón (Springer-Verlag, Berlin, 1995), pp. 749–759.
[9] M. A. Nowak, S. Bohoeffer, and R. M. May, Nature (London) **379**, 125 (1996).
[10] B. A. Huberman and N. S. Glance Proc. Natl. Acad. Sci. U.S.A. **90**, 7716 (1993).
[11] A. V. M. Herz, J. Theor. Biol. **169**, 65 (1994).
[12] M. Oliphant, in *Proceedings of the Fourth Artificial Life Workshop*, edited by R.A. Brooks and P. Maes (MIT, Cambridge, MA, 1994), pp. 349–352.
[13] G. B. Pollock Soc. Networks **11**, 175 (1989).
[14] A. Mukherji, V. Rajan, and J. R. Slagle, Nature (London) **379**, 125 (1996).
[15] G. Szabó and C. Tőke, Phys. Rev. E **58**, 69 (1997).
[16] G. R. Hardin, Science **162**, 1243 (1968).
[17] R. M. Dawes, Annu. Rev. Psychol. **31**, 169 (1980).